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Effect of two-dimensional conduction in the condensate film on laminar film condensation on a horizontal tube with variable wall temperature

Y.-Q. ZHOU† and J. W. ROSE‡

Department of Mechanical Engineering, Queen Mary and Westfield College, University of London, London E1 4NS, U.K.

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Abstract—Earlier solutions of the problem of laminar film condensation on a horizontal tube with variable surface temperature have assumed locally radial conduction across the condensate film. The present paper describes an investigation of the effects of two-dimensional conduction, inevitably present when the surface temperature of the tube is non-uniform. It was found that the simple approach is conservative and only significantly in error for $(\mu k\overline{\Delta T}/\rho\tilde{\rho}gd^3h_{fg})^{1/4} \geqslant$ about 0.01, i.e. around the highest value which might be found in practice of the dimensionless parameter. When the streamwise convection term is included in the energy equation for the condensate film, the results depend (very weakly) on the additional parameter $\rho\tilde{\rho}gd^3c_P/\mu k$, but are negligibly affected in the practical range. Copyright © 1996 Elsevier Science Ltd.

INTRODUCTION

The various improvements (boundary-layer treatments including inertia and convection terms and shear stress at the condensate surface) which, following the advent of computers, have been made to the original Nusselt [1] analysis of laminar film condensation of a 'stationary' vapour on a horizontal cylinder (see Fig. 1), have shown Nusselt's analysis for uniform surface temperature to be remarkably accurate§ for the practical ranges of the relevant parameters. The Nusselt result for the mean heat-transfer coefficient may be written

$$\overline{Nu} = 0.7280\left\{\frac{\rho\tilde{\rho}gh_{fg}d^3}{\mu k\Delta T}\right\}^{1/4}. \quad (1)$$

Noting that experiment (see for instance [2]) and conjugate (vapour-to-coolant) solutions [3] showed significant temperature variation around the tube surface during condensation, and that this could be approximated by a cosine surface temperature variation, Memory and Rose [4] solved the Nusselt problem with a prescribed surface temperature distribution

$$T_0 = a\cos\phi + \bar{T}_0. \quad (2)$$

This gives, for the vapour-to-surface temperature difference

$$\Delta T = \overline{\Delta T}(1 - A\cos\phi) \quad (3)$$

where

$$A = a/\overline{\Delta T} \quad 0 \leqslant A \leqslant 1. \quad (4)$$

With the Nusselt assumptions (apart from that of uniform surface temperature) this leads to the following equation for the local condensate film thickness:

$$\sin\phi\frac{dz}{d\phi} + \frac{4}{3}z\cos\phi - 2(1 - A\cos\phi) = 0 \quad (5)$$

where

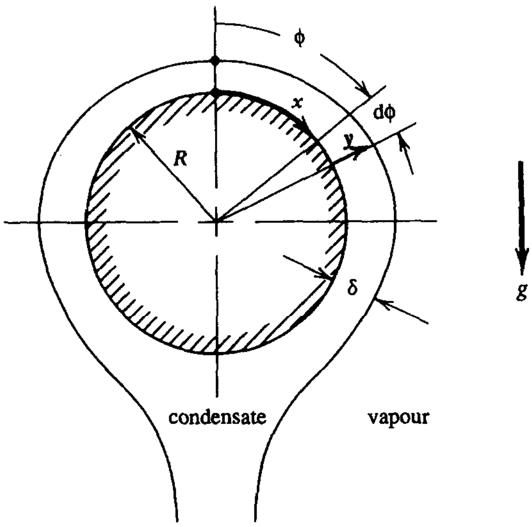

Fig. 1. Film condensation on a horizontal tube—physical model and co-ordinates.

† Nippon Otis Elevator Co. KSP 3-2-1-Sakato, Takatsu-ku, Kawasaki City, Japan.
‡ Author to whom correspondence should be addressed.
§ The accuracy of Nusselt's coefficient of 0.725, obtained using two integrations by planimetry, is also noteworthy.

NOMENCLATURE

A	defined in equation (4)
a	defined in equation (2)
B	defined in equation (17)
c_P	isobaric specific heat-capacity of condensate
d	diameter of tube
Gr	$\rho\tilde{\rho}gd^3/\mu^2$
g	specific force of gravity
h_{fg}	specific enthalpy of evaporation
k	thermal conductivity of condensate
m	local condensation mass flux
$\overline{Nu}$	Mean Nusselt number, $\bar{\alpha}d/k$
Pr	$\mu c_P/k$
q	local heat flux
q^*	dimensionless heat flux defined in equation (7)
$\bar{q}$	mean heat flux
R	radius of tube
Ra	$Gr \cdot Pr$
$\tilde{r}$	defined in equation (15)
T_0	local surface temperature of tube
$\bar{T}_0$	mean surface temperature of tube
$\tilde{T}$	defined in equation (12)
u	streamwise component of velocity in condensate film
y	radial distance from tube surface
$\tilde{y}$	defined in equation (13)
z	dimensionless local film thickness defined in equation (6).

Greek symbols

α	local heat-transfer coefficient
$\bar{\alpha}$	mean heat-transfer coefficient
ΔT	local vapour-to-surface temperature difference
$\overline{\Delta T}$	mean vapour-to-surface temperature difference
δ	local condensate film thickness
μ	viscosity of condensate
ρ	density of condensate
ρ_V	density of vapour
$\tilde{\rho}$	$\rho-\rho_V$
ϕ	angle measured from top of tube.

$$z = \left\{\frac{\rho\tilde{\rho}gh_{fg}}{\mu dk\overline{\Delta T}}\right\}\cdot \delta^4. \tag{6}$$

When a, and hence A, are zero equation (5) reduces to the Nusselt form.

Memory and Rose [4] solved equation (5), subject to the condition that z remains finite at $\phi = 0$, for values of A in the range 0–1. The dependence on ϕ of local dimensionless film thickness and heat flux differed markedly (for $A > 0$) from the Nusselt isothermal surface result. In the limiting case, $A = 1$, the dimensionless heat flux, given by

$$q^* = q\left\{\frac{\mu d}{\rho\tilde{\rho}gh_{fg}k^3\overline{\Delta T}^3}\right\}^{1/4} \tag{7}$$

increased from zero at $\phi = 0$ to a maximum of around 1.15 at $\phi \approx 2\pi/3$ before decreasing to zero at $\phi = \pi$. In the Nusselt case q^* decreases monotonically from around 0.9 at $\phi = 0$ to zero at $\phi = \pi$. However, it was found that the *mean* heat-transfer coefficient ($\bar{\alpha} = \bar{q}/\overline{\Delta T}$) for the whole tube was virtually constant and given by

$$\overline{Nu} = 0.7280\left\{\frac{\rho\tilde{\rho}gh_{fg}d^3}{\mu k\overline{\Delta T}}\right\}^{1/4} \tag{8}$$

for all values of A. Thus, even in the presence of strong surface temperature variation, the Nusselt result (with $\overline{\Delta T}$ replacing uniform ΔT) was shown to be extremely accurate for the purpose of calculating the mean heat-transfer coefficient.

Both in the solutions of Memory and Rose [4] and the conjugate analysis of Honda and Fujii [3] (who considered two-dimensional (2D) conduction in the tube wall) the local condensation mass flux at the surface of the condensate film was taken as

$$m = \frac{q}{h_{fg}} = \frac{k\Delta T}{h_{fg}\delta} \tag{9}$$

i.e. assuming locally uniform radial conduction across the thin condensate film. For the case where the outer and inner surface of the condensate film have uniform temperatures, this would be expected to give satisfactory results. The extent to which this assumption affects the results when the tube surface temperature is non-uniform is investigated in the present paper.

ANALYSIS

Two-dimensional conduction in the condensate film

Without the assumption of radial conduction in the condensate film, the condensation mass flux is given by

$$m = \frac{k}{h_{fg}}\left(\frac{\partial T}{\partial y}\right)_{y=\delta} \tag{10}$$

where y is radial distance outward from the tube surface.

The differential equation for the film thickness then becomes

$$\sin\phi\frac{dz}{d\phi}+\frac{4}{3}z\cos\phi+2\left(\frac{\partial\tilde{T}}{\partial\tilde{y}}\right)_{\tilde{y}=1}=0 \quad (11)$$

where

$$\tilde{T}=\frac{T_\infty-T}{T_\infty-T_0}=\frac{T_\infty-T}{\overline{\Delta T}} \quad (12)$$

$$\tilde{y}=\frac{y}{\delta} \quad (13)$$

and $\tilde{T}$ satisfies the conduction equation for the condensate film

$$\frac{1}{\tilde{r}}\frac{\partial}{\partial\tilde{r}}\left(\tilde{r}\frac{\partial\tilde{T}}{\partial\tilde{r}}\right)+\frac{1}{\tilde{r}^2}\frac{\partial^2\tilde{T}}{\partial\phi^2}=0 \quad (14)$$

where

$$\tilde{r}=(R+y)/R. \quad (15)$$

Uniform temperature at the condensate surface gives the boundary condition

$$\tilde{T}=0 \quad \text{at } \tilde{r}=1+\frac{\delta}{R}=1+2Bz^{1/4} \quad (16)$$

where

$$B=(\mu k\overline{\Delta T}/\rho\tilde{\rho}gd^3h_{fg})^{1/4}. \quad (17)$$

The cosine surface temperature distribution [equations (2)–(4)] used by Memory and Rose [4] gives a second boundary condition

$$\tilde{T}=1-A\cos\phi \quad \text{at } \tilde{r}=1. \quad (18)$$

In addition, by symmetry, there is no heat flux across the vertical plane through the condensate film at the top and bottom of the tube so that $\partial\tilde{T}/\partial\phi$ is zero at $\phi=0$ and $\phi=\pi$.

The following iterative procedure was adopted for solving equations (11) and (14) simultaneously:

A first estimate of $(\partial\tilde{T}/\partial\tilde{y})_{\tilde{y}=1}$ was made assuming a linear temperature drop across the condensate film as used in Memory and Rose [4], i.e.

$$(\partial\tilde{T}/\partial\tilde{y})_{\tilde{y}=1}=-(1-A\cos\phi). \quad (19)$$

Equation (11) was then solved numerically (Runge–Kutta) for a specified value of A to give a first approximation for $z(\phi)$. Equation (14) was then solved numerically (finite difference, 30×40 grid points) using the first approximation for $z(\phi)$ in boundary condition equation (16) and with a specified value of B, together with equation (18) and $\partial\tilde{T}/\partial\phi=0$ at $\phi=0$ and $\phi=\pi$. From the solution we obtain

$$\left(\frac{\partial\tilde{T}}{\partial\tilde{r}}\right)_{\tilde{r}=1+2Bz^{1/4}} \quad \text{and hence} \quad \left(\frac{\partial\tilde{T}}{\partial\tilde{y}}\right)_{\tilde{y}=1}$$

from

$$\left(\frac{\partial\tilde{T}}{\partial\tilde{y}}\right)_{\tilde{y}=1}=2Bz^{1/4}\left(\frac{\partial\tilde{T}}{\partial\tilde{r}}\right)_{\tilde{r}=1+2Bz^{1/4}}. \quad (20)$$

On the basis of the new and initial values of $(\partial\tilde{T}/\partial\tilde{y})_{\tilde{y}=1}$, and using a suitable weighting factor, a revised estimate was made for the next solution of equation (11). The process was repeated until a desired degree of convergence [of z and $(\partial\tilde{T}/\partial\tilde{y})_{\tilde{y}=1}$] was achieved. As a check on the solution, the heat-transfer rate for the whole tube, determined from the radial temperature gradient at the outside surface of the condensate film, was compared with that found using the gradient at the tube surface. The difference between

$$\int_0^\pi(1+2Bz^{1/4})\left(\frac{\partial\tilde{T}}{\partial\tilde{y}}\right)_{\tilde{y}=1}d\phi \quad \text{and} \quad \int_0^\pi\left(\frac{\partial\tilde{T}}{\partial\tilde{y}}\right)_{\tilde{y}=0}d\phi$$

was less than 1% in all cases. For values of $B\leqslant0.01$ the difference was less than 0.1% for all A.

In all cases, as found by Memory and Rose [4] for radial conduction across the condensate film, the value of $\overline{Nu}\cdot B$ [i.e. the coefficient in equation (8)] was insensitive to the value of A. The dependence of $\overline{Nu}\cdot B$ on B is summarized in Table 1. It is seen from Table 1 that only at extreme values of B is the mean Nusselt number given by equation (1) significantly in error (by around 15% at $B=0.1$) and in this case equation (1) is conservative. For the normal range of values of B ($B<0.01$) the error in the mean Nusselt number given by equation (1) is negligible.

Figures 2 and 3 compare the dimensionless film thickness and dimensionless surface heat flux with those obtained in the 1D conduction approach of Memory and Rose [1]. It is seen that for $B=0.01$ the 2D analysis leads to marginally thinner condensate films and marginally higher heat fluxes, consistent with the higher values of $\overline{Nu}\cdot B$ given in Table 1. For $B\leqslant0.01$ the present solutions for z and q^* virtually coincide with those of Memory and Rose [4].

Streamwise convection

It might be thought that, in the presence of varying surface temperature, the heat-transfer to the tube might be influenced by streamwise convection. When

Table 1. Dependence of $\overline{Nu}\cdot B$ on B

B	$\overline{Nu}\cdot B$
0.0001	0.728
0.001	0.728
0.01	0.739
0.1	0.836

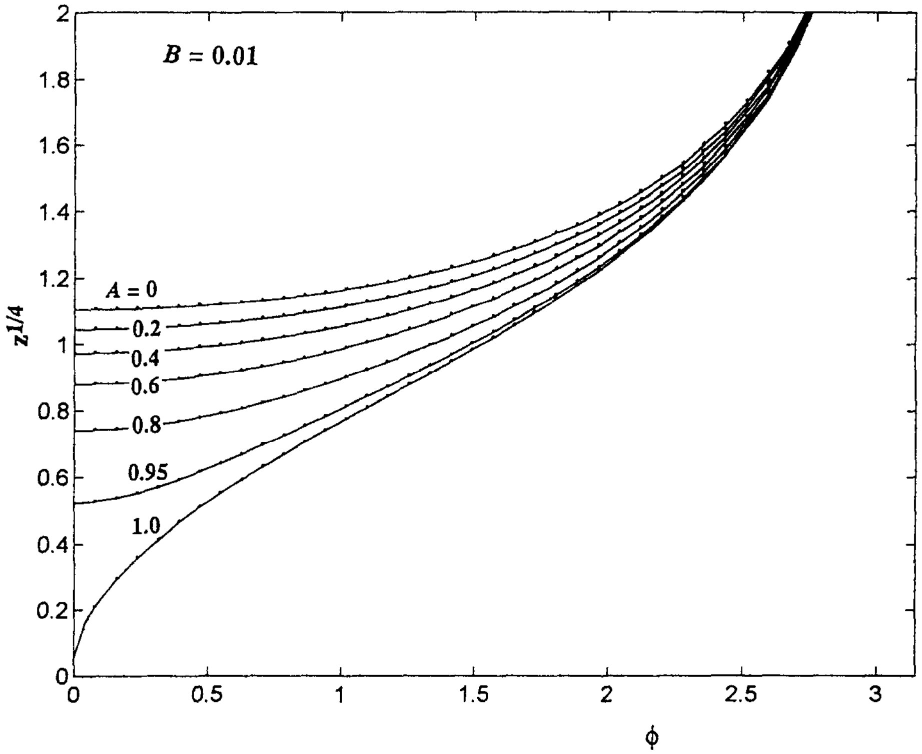

Fig. 2. Dependence of dimensionless film thickness [see equation (6)] on angle. Lines denote present results. Points denote solutions of Memory and Rose [4].

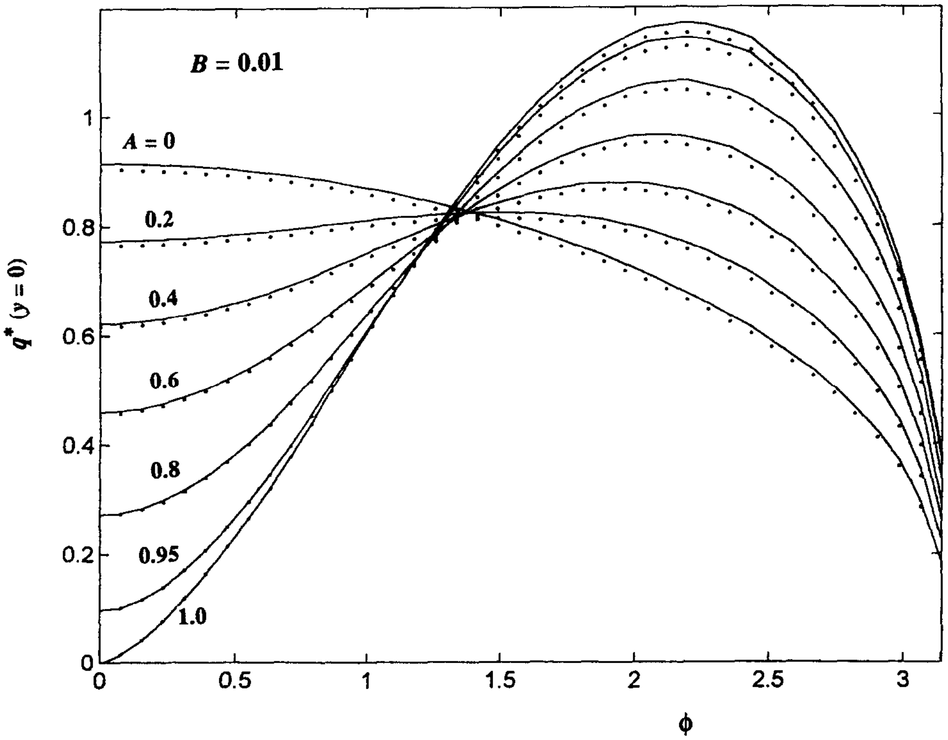

Fig. 3. Dependence of dimensionless heat flux [see equation (7)] on angle. Lines denote present results. Points denote solutions of Memory and Rose [4].

the streamwise velocity component u is included, equation (14) becomes

$$\frac{\rho c_P u R}{k}\frac{\partial \tilde{T}}{\partial \phi} = \frac{1}{\tilde{r}}\frac{\partial}{\partial \tilde{r}}\left(\tilde{r}\frac{\partial \tilde{T}}{\partial \tilde{r}}\right) + \frac{1}{\tilde{r}^2}\frac{\partial^2 \tilde{T}}{\partial \phi^2} \quad (21)$$

and

$$u = \frac{\tilde{\rho} g}{\mu}\sin\phi\left(\delta y - \frac{y^2}{2}\right). \quad (22)$$

After substituting for u from equation (22), equation (21) may be written in terms of the dimensionless variables

$$\frac{1}{\tilde{r}}\frac{\partial}{\partial \tilde{r}}\left(\tilde{r}\frac{\partial \tilde{T}}{\partial \tilde{r}}\right) + \frac{1}{\tilde{r}^2}\frac{\partial^2 \tilde{T}}{\partial \phi^2} = \frac{Ra}{16}(\tilde{r}-1)(4Bz^{1/4} - \tilde{r} + 1)\sin\phi\frac{\partial \tilde{T}}{\partial \phi} \quad (23)$$

where

$$Ra = Gr \cdot Pr = \rho\tilde{\rho} g d^3 c_P/\mu k. \quad (24)$$

Evidently streamwise convection becomes less important for small values of Ra. In practice Ra is generally in the range 10^7–10^{12}. Solutions were obtained as before using equation (23) in place of equation (14) for values of Ra up to 10^{15}. The results obtained differed negligibly from those obtained earlier.

CONCLUSION

Even in the presence of strong surface temperature variation, the 1D conduction approximation for the condensate film leads to negligible error in the practical range of the parameter $B = (\mu k \Delta T/\rho\tilde{\rho} g d^3 h_{fg})^{1/4}$. For extreme values of B the 1D analysis is conservative. Streamwise convection has negligible effect on the heat transfer for the practical ranges of B and Ra.

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